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ABSTRACT

A new one-semester course in geometry was designed, less axiomatic in structure yet including content traditionally contained in a first course in geometry. Conducting original investigations, constructing definitions, developing arguments to explain relationships, and creating resource materials are additional objectives, with many experiences in drawing, measuring, manipulating, exploring, and investigating. Evaluation of the course involved three classes; data from 75 students indicated that they achieved at least as well as normative groups on the Cooperative Mathematics Geometry Test. (MS)

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A REPORT ON A TESTED ALTERNATIVE TO THE TRADITIONAL HIGH SCHOOL GEOMETRY COURSE

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A REPORT ON A TESTED ALTERNATIVE TO THE TRADITIONAL HIGH SCHOOL GEOMETRY COURSE

Introduction

There has been a great deal of controversy concerning the nature and content of the high school geometry course which is normally a part of the college preparatory mathematics sequence. Some educators support the Euclidean synthetic approach; others a transformation approach; others a coordinate approach; others an affine approach; others a vector approach; and some support an integration of several of the above. On the other hand, some educators hold that Geometry should not be taught as a separate course but should be integrated with other mathematical experiences of the high school student.

The nature of the disagreement concerning high school geometry is illustrated by the following statements. Moise (7) has written, "Most of the material is so fundamental that ignorance of it ought to be regarded as unthinkable. . . . Geometry seems to be the only mathematical subject that young students can understand and work with in approximately the same way as a mathematician" (pg. 475). Fehr (4) in opposition states, "Euclid's synthetic treatment of his 'space' is simply out of the mainstream of current mathematical thinking and is seldom used by pure or applied mathematicians. As a year of formal mathematical study, it must depart" (pg. 373).

One of the major areas of controversy concerning many educators is the degree of axiomatic emphasis of traditional geometry courses. Significant attempts have been made to strengthen the axiomatic structure of high school geometry. Current textbooks in wide use are products of such efforts and are based on the School Mathematics Study Group project. The traditional geometry course has had as its primary objective the development of the entire body of course content as a unified abstract mathematical system. Moise (7) in support of the Euclidean, axiomatic approach writes, "The search for coherence and order over and above the search for new facts is a vital part of scientific thought. . . . I think it is the only example that young students are likely to understand" (pg. 476).

On the other hand, proponents of less axiomatic emphasis in high school geometry hold with Brumfiel (2) that concern with axiomatics, logic, and formal proof slows the development of geometry and actually has only superficial impact on students. As the result of investigations

over a period of years, Brumfiel (2) concluded that students acquire little understanding of geometric structure as a result of contemporary axiomatic treatments of geometry. He goes so far as to say, "Whatever axioms are printed in the text, students will not be disturbed, for they will cheerfully ignore them all" (pg. 99). Forbes (5) concludes an argument against axiomatic emphasis with: "The student is engaged in a year-long task of the construction of an enormous complicated structure as his first try at coping with an abstract system" (pg. 337). Fehr (4) does not agree with the position that geometry is the only subject available at the secondary level to introduce an axiomatic development. He states, "Recent advances in Algebra, Probability Theory and Analysis have made it possible to use these topics in an elementary and simple manner to introduce axiomatic structure" (pg. 151).

In light of this controversy we began to examine the geometry course offered at P. K. Yonge.

The Need For An Alternative Geometry Course

The Geometry course offered at P. K. Yonge was a traditional, Euclidean, year-long, textbook-oriented course with emphasis on formal axiomatic structure and proof construction. The textbook was one based on SMSG materials.

Increasing numbers of our students were experiencing difficulty with this geometry course. The major source of difficulty seemed to be the formal axiomatic structure of the course. The rigor required for initial attempts of proof construction for statements that students perceived as obvious was also a source of difficulty. An additional problem that resulted from difficulty with this aspect of the course was that many students were not acquiring the necessary body of concepts and skills traditionally contained in a first course in geometry. These concerns caused us to consider the possibility of alternatives to the existing geometry course.

An additional concern caused us to consider alternatives to the year-long geometry course. As part of a four-year college preparatory sequence of Algebra I, Geometry, Algebra II, and Advanced Mathematics, students experienced an absence of a full year from the study of algebra. Consequently, our Algebra II students needed a great deal of review of Algebra I before they were ready to proceed with the new material of Algebra II. In order to minimize the need for review time in Algebra II, we decided to limit our new geometry course to one semester and follow it with a second semester course entitled Intermediate Algebra. Hence, the four-year sequence then became Algebra I,

Geometry/Intermediate Algebra, Algebra II, and Advanced Mathematics. An immediate question that resulted from this new sequence concerned the effect that the shorter length of the course would have on the geometry achievement of our students.

The P. K. Yonge Geometry Course

The new shorter course in geometry was designed in general, to be less axiomatic in structure yet to include the geometric skills and concepts traditionally contained in a first course of geometry. It was desired that the new course would provide opportunities for the great majority of students enrolled to experience success while, specifically, causing students to:

1. Acquire the skills and concepts traditionally found in a first year geometry course;
2. conduct original investigations leading to the stating and testing of generalizations;
3. recognize the need for definitions and to participate in the construction of acceptable definitions;
4. use definitions and generalizations in the construction of convincing arguments for the purpose of explaining specific relationships usually proved in a first year geometry course; and
5. participate in the creation and development of their own resource materials.

Many experiences in drawing, measuring, manipulating, exploring, and investigating were provided in order to accomplish these objectives. Significant results of these activities were definitions and generalizations which students recorded in notebooks, the primary resource for study and review. We considered it important to involve students in the development of geometric concepts rather than to give them a textbook that contained a previously determined structure and set of concepts. As we provided experiences in the construction of proofs, we tried to avoid making initial demands that might interfere with original and creative construction of a convincing argument. Our previous experience had caused us to believe that initial emphasis on formal structure sometimes caused students to be unsuccessful in creating proofs. Students were encouraged to write informal explanations that were clear and convincing to classmates and teacher. A higher degree of rigor was expected of the student as the course progressed and as the students became more experienced in the construction of arguments.

The new geometry course was taught for the first time at P. K. Yonge during the 1974-75 school year. Initial outlines and plans for the new course were accomplished during two weeks of summer planning time. Modifications were made as a result of experiences in teaching the course. The final course outline consisted of six units as follows:

Unit 1. Points, Lines, and Planes

Unit 2. Triangles

Unit 3. Parallel Lines and Quadrilaterals

Unit 4. Area and Special Triangle Relationships

Unit 5. Similarity

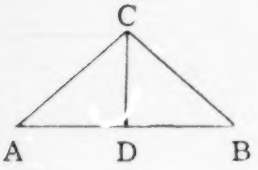
Unit 6. Circles

The plan for each of the six units included a list of terms to be defined, symbols, generalizations, and activities for each unit. The first unit began with the undefined terms of point, line, and plane and proceeded to define and classify other terms. Most of the activities were drawing, measuring, and observing relationships. For example, after all the terms in Unit 1 were defined and discussed, this exercise was given to students.

> Draw two lines that intersect in one point and find the measures of all the angles. Draw three lines that intersect in exactly one point and find the measures of all the angles. State any relationships among the angles.

From this exercise most students were able to generalize that vertical angles are congruent and supplements of the same or congruent angles are congruent. Some students were dependent on measurement to see these relationships, and many students were able to explain algebraically the second generalization.

In the discussion of triangle congruency relationships in Unit 2, drawing and measuring exercises convinced students that SAS $\tilde{=}$ SAS, ASA $\tilde{=}$ ASA, and SSS $\tilde{=}$ SSS are always congruency correspondences, but SSA $\tilde{=}$ SSA and AAA $\tilde{=}$ AAA are not always congruency correspondences. By the fourth week of the semester, most students were able to construct convincing arguments like the one below.



In isosceles ΔABC, $\overline{CD}$ is the median to the base. Why is $\Delta ADC \cong \Delta BDC$?

ΔABC is isosceles, means that $\overline{AC} \cong \overline{BC}$. $\overline{CD}$ is the median means that D is the mid-point of $\overline{AB}$, so $\overline{AD} \cong \overline{BD}$. $\angle A \cong \angle B$ because the base angles of an isosceles Δ are $\cong$. . . $\Delta ADC \cong \Delta BDC$ because SAS $\cong$ SAS.

Most students understood these arguments and regarded the problems as ones which were not obvious but in need of explanations. As the course progressed, students were asked to construct arguments for more difficult questions. The structure of the arguments was similar to the example given, but the degree of difficulty of the problems and the degree of completeness of the arguments were increased. Formal axiomatic structure was not part of the course. A set of previously determined assumptions, which could have provided the logical foundation for student arguments, was not included in order to decrease the difficulty of initial argument construction.

Evaluation

Ninety-three students were enrolled in the P. K. Yonge semester-length geometry course. Most of the students were in the tenth grade, but a few ninth grade accelerated students were enrolled in the course. These students were distributed among three classes, each taught by one or the other of the investigators.

Geometry achievement as measured by Form A of the Cooperative Mathematics Geometry Test, Parts I and II, was assessed with a pretest and posttest procedure. The two forty-minute parts of the test were administered on successive days at the beginning and end of the course. Algebra I grades were recorded for students when available in order to examine the relationship between this grade and geometry achievement.

Eighty-eight of the students enrolled successfully completed the course. Posttest scores were not available for the five students who dropped it. Absences prevented an additional eight students from taking some portion of the pretest or posttest. In addition, Algebra I grades were not available for five of the students enrolled. A total of seventy-five students had all of the required data available for analysis.

The mean raw scores and corresponding mid-percentile ranks that resulted from the achievement tests are contained in the tables below. Part I of the test measures direct application of geometric facts and important concepts of a typical high school course, and the total score of Parts I and II is more a measure of reasoning, problem solving, and application in three-dimensional space.

PART I Mean Raw Scores

	N	Pre	%*	Post	%
Total group	(75)	17.08	8	25.92	50
Alg I A	(36)	19.53	16	29.19	72
Alg I B	(19)	16.68	6	24.95	43
Alg I C	(20)	13.05	4	20.95	20

TOTAL (Parts I and II) Mean Raw Scores

	N	Pre	%	Post	%
Total group	(75)	26.72	9	40.44	51
Alg I A	(36)	30.11	17	46.14	75
Alg I B	(19)	26.58	9	38.05	43
Alg I C	(20)	20.75	3	32.45	22

* This symbol will be used to represent the mid-percentile rank of year-long geometry students of the national norm sample.

A two-way analysis of variance with repeated measures was conducted for the Part I and Total (Part I and Part II) raw scores in order to examine the difference between the pretest scores and posttest scores of the total group and to examine the difference in gain of the three groups of students identified by previous Algebra I grades. The total group pretest and posttest mean raw scores for both Part I and the Total (Part I and Part II) were found to be significantly different. The F-ratios for Part I and Total were 22.37 and 24.58, respectively, for 2 and 72 degrees of freedom ($p \leq 0.05$). The large difference between the pretest and posttest scores is readily apparent when the corresponding percentile ranks are examined.

The difference in Part I pretest and posttest mean raw scores was significant only between the Algebra I-A group and the Algebra I-C group. The Total (Part I and Part II) mean raw score of the Algebra I-A group was significantly different from the mean raw scores of both the Algebra I-B and Algebra I-C groups.

There was a significant interaction between Algebra I group designation and time for the Total (Part I and Part II) score. The F-ratio was 3.32 for 2 and 72 degrees of freedom ($p < 0.05$). The gain of the Algebra I-A group was significantly greater than the gain of the Algebra I-B and Algebra I-C groups.

An examination of the confidence interval for the means of all four of the student groups indicated that all of the mean raw scores would consistently occur in the normative interval indicated.

The analysis of the data seems to indicate that the achievement of the students enrolled in the semester-length course was not lower than that of the normative group of students who experienced a traditional year-long geometry course. The mid-percentile ranks corresponding to the mean raw scores of the Algebra I-A student group indicate that this course was certainly adequate for more capable students. The large number of students who completed the course is an indication that we were able to structure the course in such a way that a notable majority of students could be successful. However, the efforts to retain less successful students resulted in a lower total group gain. If the posttest mean raw scores are determined for Algebra I-A and Algebra I-B students only, higher means result. Mean raw scores of 27.73 for Part I and 43.34 for the Total (Part I and Part II) are obtained. These scores correspond to mid-percentile ranks of 57 and 60, respectively.

Student opinion was solicited during the course. Student opinion was mixed concerning the absence of a textbook. Some students felt that their own resources created during the course were not as satisfactory as they thought a textbook would be. Other students indicated much pleasure with the increased involvement in the creation of resource materials. Responses were generally positive about the exploratory approach and also about the process of constructing convincing arguments and organizing the results systematically. A period of adjustment was necessary for all students as the absence of a text and emphasis on student exploration in a mathematics class were new experiences for them.

Implications

The results of the evaluation of the geometry course have caused us to feel that it is proper to continue our current college preparatory sequence. It appears that we can afford to shorten the initial geometry experience to one semester in order to provide time for an algebra experience during that year for the students. However, we are of the opinion that we would use a similar non-axiomatic, exploratory approach for our geometry course if it were a year-long course. We recognize the logical weakness of the absence of a foundation provided by a formal axiomatic structure. We feel that the students' experiences in constructing deductive arguments were not weakened by the absence of postulates, nor do we believe that students' acquisition of skills and concepts or ability to construct a proof would have been enhanced by their attempts to understand the position and significance of postulates in an axiomatic structure. In fact, it is our opinion that students' willingness and ability to construct a convincing argument were increased by the absence of concern about postulates.

We do feel that axiomatic structure is important and that it should be included as part of the high school mathematics experience. We plan to include axiomatic investigation later in the high school sequence when most students appear ready to grasp the concept of structure and relate it to the material that they are currently examining.

As the result of our experience with the creation of a new geometry course, a school or school system with similarly perceived needs should be less reluctant to attempt the construction of such a course. We particularly feel that such a course would be appropriate for smaller high schools that are not able to offer several levels of difficulty of college preparatory geometry. We would like to emphasize that the non-axiomatic aspect of the structure of the course was more important than the change in the length of the course.

As we began to teach the course, we were concerned about how we would have to cope with the ways in which the new course was different from the traditional geometry course we had taught previously. The absence of a textbook for a problems source was of a particular concern as well as the increased preparation effort required. However, the second time we taught the course, the preparation demands did not seem so great, and we used problems which we had collected the previous year. The increased student involvement and the absence of the usual amount of student frustration made the effort more than worthwhile. We would encourage others to consider such an undertaking seriously.

Bibliography

1. Anderson, Richard D., et al. School Mathematics Geometry. Boston: Houghton Mifflin Company. 1969.

2. Brumfiel, Charles. "Conventional Approaches Using Synthetic Euclidean Geometry", Geometry in the Mathematics Curriculum. Thirty-sixth Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: The National Council of Teachers of Mathematics, Inc. 1973.

3. Fehr, Howard F. "The Present Year-Long Course in Euclidean Geometry Must Go", The Mathematics Teacher. 65:102. February, 1972.

4. Fehr, Howard F. "Geometry as a Secondary School Subject", Geometry in the Mathematics Curriculum. Thirty-sixth Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: The National Council of Teachers of Mathematics, Inc. 1973.

5. Forbes, Jack E. "An Eclectic Program in Geometry", Geometry in the Mathematics Curriculum. Thirty-sixth Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: The National Council of Teachers of Mathematics, Inc. 1973.

6. Moise, Edwin E. and Floyd L. Downs, Jr. Geometry. Menlo Park, California: Addison-Wesley. 1967.

7. Moise, Edward E. "The Meaning of Euclidean Geometry in School Mathematics", The Mathematics Teacher. 68:472-477. October, 1975.

APPENDIX

Unit 1 and Unit 4 of the Course Outline

UNIT 1

Point, Line, and Plane

This unit will introduce the student to exploratory nature of the course. The starting place will be the undefined terms: point, line, and plane. The initial objective will be for students to define terms, classify terms, and to seek relationships. The first unit will include the students' first experience with proof. Measurement will be an integral part of the students' exploration.

Terms That May Be Defined

Distance

Between

Segment Measure of Segment Midpoint Congruent Segment

Bisect

Ray Opposite Rays

Collinear

Angle Vertex of angle, side, interior, exterior, measure of angle, congruent angles, acute, obtuse, supplementary, complementary, vertical angles, adjacent angles

Linear pair

Perpendicular Line, ray, segment, plane

Parallel Line, ray segment, plane

Skew Lines

GENERALIZATIONS

1. Two points determine a unique line.
2. If two lines intersect, their intersection is a point.
3. If two planes intersect, their intersection is a line.
4. The number of lines determined by n points.
5. Three non-collinear points determine a unique plane.
6. If two angles form a linear pair, they are supplementary.
7. Vertical angles are congruent.
8. Supplements of congruent angles are congruent.
9. Complements of congruent angles are congruent.
10. If two angles are complementary, they are both acute.
11. If two angles are supplementary and congruent, they are both right angles.
12. The line perpendicular to a line through a point on the line is unique.
13. The line perpendicular to a line through a point not on the line is unique.
14. Lines perpendicular to the same line and coplanar are parallel.

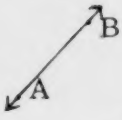
ACTIVITIES AND OBJECTIVES

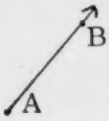
1. Draw and identify representations of all sets of points defined.
2. Draw representations of all relationships determined.
3. Measure all sets of points that are measurable (English & Metric).
4. Exploration of relationships through drawing and measuring.
5. Solutions of algebraic problems that occur through the use of relationships.
6. Pencil and paper treasure hunt.

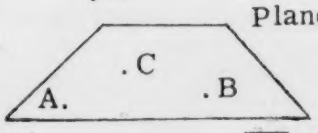
Symbols

(will be dittoed and put in notebook)

Point . B B

Line  $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

Ray  $\overrightarrow{AB}$

Plane  Plane ABC

Segment $\overline{AB}$ or $\overline{BA}$

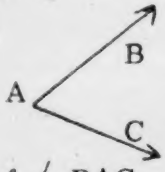
Measure of Segment $m\overline{AB}$ AB

Congruent $\cong$

Perpendicular $\perp$

Parallel $\parallel$

Right Angle

Angle  $\angle BAC$ or $\angle A$

Measure of $\angle$ BAC $m\angle BAC$

UNIT 4

Area and Special Triangle Relationships

Terms to be defined:

Area

Generalizations:

Area of Square
Rectangle
Parallelogram
Triangle
Trapezoid

Pythagorean Theorem and converse

30-60-90 Triangle relationships

Isosceles right triangle relationships

Activities and Objectives:

1. Prove all generalizations except square area.
2. Compute area of squares, rectangles, etc. and composite figures.
3. Construct segments whose measure is $\sqrt{2}$, $\sqrt{3}$, etc.